Association rule mining

Philippe Fournier-Viger
http://www.philippe-Fournier-viger.com


Source code and datasets available in the SPMF library
Association rule mining

- A data analysis task proposed by Agrawal & Srikant (1994)
- **Goal:** find interesting associations between values in a dataset.
- E.g. discover the products that people like to purchase together frequently

- In this video, I will explain “association rule mining” and the relationship with “itemset mining”
ITEMSET MINING (BRIEF REVIEW)
Frequent itemset mining (频繁项集挖掘)

A transaction database:

<table>
<thead>
<tr>
<th>Transaction</th>
<th>Items appearing in the transaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>{pasta, lemon, bread, orange}</td>
</tr>
<tr>
<td>T2</td>
<td>{pasta, lemon}</td>
</tr>
<tr>
<td>T3</td>
<td>{pasta, orange, cake}</td>
</tr>
<tr>
<td>T4</td>
<td>{pasta, lemon, orange cake}</td>
</tr>
</tbody>
</table>

For \( \text{minsup} = 2 \), the frequent itemsets are:

\{lemon\}, \{pasta\}, \{orange\}, \{cake\}, \{lemon, pasta\}, \{lemon, orange\}, \{pasta, orange\}, \{pasta, cake\}, \{orange, cake\}, \{lemon, pasta, orange\}
\textbf{minsup} = 2

frequent itemsets

Infrequent itemsets

\begin{align*}
l & = \text{lemon} \\
p & = \text{pasta} \\
b & = \text{bread} \\
o & = \text{orange} \\
c & = \text{cake}
\end{align*}
Property 2: Let there be an itemset $Y$. If there exists an itemset $X \subseteq Y$ such that $X$ is infrequent, then $Y$ is infrequent.

Example:
• Consider $\{\text{bread, lemon}\}$.
• If we know that $\{\text{bread}\}$ is infrequent, then we can infer that $\{\text{bread, lemon}\}$ is also infrequent.

<table>
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</thead>
<tbody>
<tr>
<td>T1</td>
<td>${\text{pasta, lemon, bread, orange}}$</td>
</tr>
<tr>
<td>T2</td>
<td>${\text{pasta, lemon}}$</td>
</tr>
<tr>
<td>T3</td>
<td>${\text{pasta, orange, cake}}$</td>
</tr>
<tr>
<td>T4</td>
<td>${\text{pasta, lemon, orange, cake}}$</td>
</tr>
</tbody>
</table>
This property is useful to reduce the search space.

Example:

\[ \text{minsup} = 2 \]

If « bread » is infrequent
This property is useful to reduce the search space.

Example:

If « bread » is infrequent, all its supersets are infrequent.
ASSOCIATION RULE MINING
(关联规则挖掘)
Introduction

- Finding frequent patterns in a database allows to find useful information.
- But it has some limitations
A transactional database $D$

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</tr>
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</table>

If $\text{minsup} = 2$, then $\{\text{pasta, cake}\}$ is frequent.

Can we conclude that people who buy pasta will also buy cakes?
An **association rule** is a rule of the form $X \rightarrow Y$ where

- $X$ and $Y$ are itemsets,
- and $X \cap Y = \emptyset$.

**e.g.**

- $\{\text{orange, cake}\} \rightarrow \{\text{pasta}\}$
- $\{\text{lemon, orange}\} \rightarrow \{\text{pasta}\}$
- $\{\text{pasta}\} \rightarrow \{\text{bread}\}$
- ...
The *support of a rule* $X \rightarrow Y$ is calculated as $\text{sup}(X \rightarrow Y) = \frac{\text{sup}(XUY)}{|D|}$ where $|D|$ is the number of transactions.

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</table>

**e.g.** \{lemon, orange\} $\rightarrow$ \{pasta\} has a support of 0.5 i.e. two out of four transactions.
The **confidence of a rule** \( X \rightarrow Y \) is calculated as

\[
\text{conf}(X \rightarrow Y) = \frac{\text{sup}(XUY)}{\text{sup}(X)}.
\]

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\{lemon, orange\} \rightarrow \{pasta\} has a confidence of 1.0 (100%)
The **confidence of a rule** $X \rightarrow Y$ is calculated as
\[
\text{conf}(X \rightarrow Y) = \frac{\text{sup}(XUY)}{\text{sup}(X)}.
\]

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\{pasta\} $\rightarrow$ \{lemon\} has a confidence of 0.75
\{lemon\} $\rightarrow$ \{pasta\} has a confidence of 1.0
Association rule mining

Input:
- A transaction database (set of transactions)
- A parameter $\text{minsup}$ ($0 \geq \text{minsup} \geq 1$)
- A parameter $\text{minconf}$ ($0 \geq \text{minconf} \geq 1$)

Output: each association rule $X \rightarrow Y$ such that:
- $\text{sup}(X \rightarrow Y) \geq \text{minsup}$ and
- $\text{conf}(X \rightarrow Y) \geq \text{minconf}$

{pasta} $\rightarrow$ {lemon} has a confidence of 0.75
{lemon} $\rightarrow$ {pasta} has a confidence of 1.0
Example

\[
\text{minsupt} = 0.5 \quad \text{minconf} = 0.75
\]

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</table>

- \text{llem} \rightarrow \text{pasta} \\
  support: 3 \\
  confidence: 1
- \text{pasta} \rightarrow \text{lemon} \\
  support: 3 \\
  confidence: 0.75
- \text{orange} \rightarrow \text{pasta} \\
  support: 3 \\
  confidence: 1
- \text{pasta} \rightarrow \text{orange} \\
  support: 3 \\
  confidence: 0.75
- \text{cake} \rightarrow \text{pasta} \\
  support: 2 \\
  confidence: 1
- \text{cake} \rightarrow \text{orange} \\
  support: 2 \\
  confidence: 1
- \text{llem} \text{orange} \rightarrow \text{pasta} \\
  support: 2 \\
  confidence: 1
- \text{orange} \text{cake} \rightarrow \text{pasta} \\
  support: 2 \\
  confidence: 1
- \text{pasta} \text{cake} \rightarrow \text{orange} \\
  support: 2 \\
  confidence: 1
- \text{cake} \rightarrow \text{pasta} \text{orange} \\
  support: 2 \\
  confidence: 1
Why using the support and confidence?

- **The support** allows to:
  - find patterns that are less likely to be random.
  - reduce the number of patterns,
  - make the algorithms more efficient.

- **The confidence** allows to:
  - measure the strength of associations
  - obtain an estimation of the conditional probability $P(Y \mid X)$.
  - **Warning**: a strong association does not mean that there is causality!
How to find the association rules?

Naïve approach

1. Create all association rules.
2. Calculate their confidence and support by scanning the database.
3. Keep only the valid rules.

This approach is inefficient. For \( d \) items, there are:

\[
\sum_{k=1}^{d-1} \binom{d}{k} \times \sum_{j=1}^{d-k} \binom{d-k}{j} = 3^d - 2^d + 1
\]

For \( d = 6 \), this means 602 rules!
For \( d = 100 \), this means \( 10^{47} \) rules!
Observation 1

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- lemon $\rightarrow$ pasta  
  support: 3  confidence: 1
- pasta $\rightarrow$ lemon  
  support: 3  confidence: 0.75
- orange $\rightarrow$ pasta  
  support: 3  confidence: 1
- pasta $\rightarrow$ orange  
  support: 3  confidence: 0.75

Observation 1. All the rules containing the same items can be viewed as having been derived from a same frequent itemset.

e.g. {pasta, lemon}
Observation 2

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- lemon → pasta  
  support: 3  
  confidence: 1
- pasta → lemon  
  support: 3  
  confidence: 0.75
- orange → pasta  
  support: 3  
  confidence: 1
- pasta → orange  
  support: 3  
  confidence: 0.75

Observation 2. All the rules containing the same items have the same support, but may not have the same confidence. e.g. {pasta, lemon}
Observation 3

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</tr>
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</tr>
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</table>

- lemon → pasta  support: 3  confidence: 1
- pasta → lemon  support: 3  confidence: 0.75
- orange → pasta support: 3  confidence: 1
- pasta → orange support: 3  confidence: 0.75

Observation 3. If an itemset is infrequent, all rules derived from that itemset can be ignored.  
**e.g.** If $\text{minsup} = 4$..., rules derived from \{pasta, lemon\} can be ignored, since its support is 3.
How to find association rules efficiently?


**Two steps:**

1. Discover the frequent itemsets.
2. Use the frequent itemsets to generate association rules having a confidence greater or equal to \( \text{minconf} \).

Step 1 is the most difficult. Thus, most studies are on improving the efficiency of Step 1.
Generating rules

- Each frequent itemset $Y$ of size $k$ can produce $2^k - 2$ rules.
- A rule can be created by dividing an itemset $Y$ in two non empty subsets to obtain a rule $X \rightarrow Y \setminus X$.
- Then, the confidence of the rule must be calculated.
Generating rules

Example: using the itemset $X=\{a, b, c\}$, we can generate:

- $\{a, b\} \rightarrow \{c\}$
- $\{a, c\} \rightarrow \{b\}$
- $\{b, c\} \rightarrow \{a\}$
- $\{a\} \rightarrow \{b, c\}$
- $\{b\} \rightarrow \{a, c\}$
- $\{c\} \rightarrow \{a, b\}$
Calculating the confidence

**Example**: using the itemset $X=\{a, b, c\}$, we can generate:

- $\{a, b\} \rightarrow \{c\}$
- $\{a, c\} \rightarrow \{b\}$
- $\{b, c\} \rightarrow \{a\}$
- $\{a\} \rightarrow \{b, c\}$
- $\{b\} \rightarrow \{a, c\}$
- $\{c\} \rightarrow \{a, b\}$

How can we calculate the confidence of rules derived from $X$?

- We must know the support of all subsets of $X$.
- We know it already, since if $X$ is a frequent itemset, then all its subsets are frequent!
Calculating the confidence

The result of a frequent itemset mining program looks like this:

{pasta} support = 4
{lemon} support = 3
{orange} support = 3
{cake} support = 2

{pasta, lemon} support: 3
{pasta, orange} support: 3
{pasta, cake} support: 2
{lemon, orange} support: 2
{orange, cake} support: 2

{pasta, lemon, orange} support: 2
{pasta, orange, cake} support: 2

How can we quickly search for the support of a set?
Calculating the confidence

Solution 1:

- Itemsets are grouped by size,
- Itemsets having the same size are sorted by some total order $\succ$.
- binary search…

\[
\begin{align*}
\{\text{pasta, lemon}\} & \quad \text{support: 3} \\
\{\text{pasta, orange}\} & \quad \text{support: 3} \\
\{\text{pasta, cake}\} & \quad \text{support: 2} \\
\{\text{lemon, orange}\} & \quad \text{support: 2} \\
\{\text{orange, cake}\} & \quad \text{support: 2}
\end{align*}
\]

pasta $\succ$ lemon $\succ$ bread $\succ$ orange $\succ$ cake
Calculating the confidence

Solution 2:

- itemsets are stored in a « trie » to search for itemsets in $O(1)$ time.

This path in the tree represents the itemset \{pasta\} which has a support of 4.
Calculating the confidence

Solution 2:

- itemsets are stored in a « trie » to search for itemsets in $O(1)$ time

This path in the tree represents the itemset \{pasta, orange\} which has a support of 3
Calculating the confidence

Solution 2:

- itemsets are stored in a « trie » to search for itemsets in \( O(1) \) time

This path in the tree represents the itemset \{pasta, orange, cake\} which has a support of 2
Reducing the search space

Can we reduce the search space using the confidence measure?

- Confidence is not an anti-monotone measure.
- However, the following relationship between two rules can be proved:

**Theorem:** If a rule $X \rightarrow Y \setminus X$ does not satisfy the confidence threshold, any rule $X' \rightarrow Y \setminus X'$ such that $X' \subseteq X$ will also not satisfy the confidence threshold.
Proof

Let there be two rules
\( X \rightarrow Y - X \) and
\( X' \rightarrow Y - X' \) such that \( X' \subseteq X \).

The confidence of these rules are

- \( \text{conf}(X \rightarrow Y - X) = \frac{\text{sup}(XUY)}{\text{sup}(X)} \)
- \( \text{conf}(X' \rightarrow Y - X') = \frac{\text{sup}(XUY)}{\text{sup}(X')} \)

Since \( X' \subseteq X \), it follows that \( \text{sup}(X') \geq \text{sup}(X) \).

Thus: \( \text{conf}(X' \rightarrow Y - X') \leq \text{conf}(X \rightarrow Y - X) \)
and the theorem holds.
Illustration

Low-confidence rule

These rules can be eliminated
Generating rules

Algorithm 8.6: Algorithm ASSOCIATIONRules

ASSOCIATIONRules \((\mathcal{F}, \ minconf)\):

1. foreach \(Z \in \mathcal{F}, \) such that \(|Z| \geq 2\) do
2. \(\{\mathcal{A} \leftarrow \{X \mid X \subset Z, X \neq \emptyset\}\}
3. while \(\mathcal{A} \neq \emptyset\) do
4. \(X \leftarrow \) le plus grand itemset dans \(\mathcal{A}\)
5. \(\mathcal{A} \leftarrow \mathcal{A} \setminus X /\!\!/ \) enlever \(X\) de \(\mathcal{A}\)
6. \(c \leftarrow \sup(Z) / \sup(X)\)
7. if \(c \geq \minconf\) then
8. \ print \(X \rightarrow Y, \sup(Z), c\)
9. else
10. \(\mathcal{A} \leftarrow \mathcal{A} \setminus \{W \mid W \subset X\} /\!\!/ \) remove all subsets of \(X\) from \(\mathcal{A}\)

where \(\mathcal{F}\) is the set of all frequent itemsets
\(\mathcal{A}\) is the set of all proper non empty subsets of \(\mathcal{F}\)
EVALUATING ASSOCIATIONS
Evaluating associations

- A large amount of patterns can be discovered
- **How to find the most interesting patterns?**
- Interestingness measures:
  - **objective measures**: statistical reasons for selecting patterns
  - **subjectives**: discover surprising or interesting patterns (e.g. *diaper → beer* is more surprising than *mouse → keyboard*).
- It is more difficult to consider subjective measures in the search for patterns.
Objective measure

- Independent from any domain
- e.g. *support* and *confidence*
- Several objective measures can be calculated using a contingency table.
  e.g. a table with 2 binary attributes

<table>
<thead>
<tr>
<th></th>
<th>$B$</th>
<th>$\bar{B}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A$</td>
<td>$f_{11}$</td>
<td>$f_{10}$</td>
</tr>
<tr>
<td>$\bar{A}$</td>
<td>$f_{01}$</td>
<td>$f_{00}$</td>
</tr>
<tr>
<td></td>
<td>$f_{+1}$</td>
<td>$f_{+0}$</td>
</tr>
</tbody>
</table>
Limitations of the support and confidence

If we use the \textit{minsup} threshold,

- we will find less results,
- it will be faster,
- but we may eliminate some rare patterns that are interesting.
Another problem

• Consider: \{tea\} \rightarrow \{coffee\}
  support : 15% confidence : 75%

• This seems like an interesting pattern…

<table>
<thead>
<tr>
<th></th>
<th>Coffee</th>
<th>\overline{Coffee}</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>\text{Tea}</td>
<td>150</td>
<td>50</td>
<td>200</td>
</tr>
<tr>
<td>\overline{\text{Tea}}</td>
<td>650</td>
<td>150</td>
<td>800</td>
</tr>
<tr>
<td></td>
<td>800</td>
<td>200</td>
<td>1000</td>
</tr>
</tbody>
</table>

\[ \text{conf}(X \rightarrow Y) = \frac{\text{sup}(X \cup Y)}{\text{sup}(X)} \]

• However, 80% of the people drink coffee no matter if they drink tea or not.
• In fact, the probability of drinking coffee for tea drinkers is lower than for non tea drinkers (80 instead of 75%).
• This problem occurs because the confidence measure does not consider the support of the left side of rules.
The lift

- \( \text{lift}(X \to Y) = \frac{\text{conf}(X \cup Y)}{\text{sup}(Y)} = \frac{\text{sup}(X \cup Y)}{\text{sup}(X) \times \text{sup}(Y)} \)

- if \( \leq 1 \), \( X \) and \( Y \) are independent
- if \( > 1 \), \( X \) and \( Y \) are positively correlated
- if \( < 1 \), \( X \) and \( Y \) are negatively correlated

Example:

\( \text{lift}({\text{tea}} \to {\text{coffee}}) = 0.9375 \),

This indicates a slightly negative correlation
Limitations of the lift

Example:

<table>
<thead>
<tr>
<th></th>
<th>𝑝</th>
<th>̅𝑝</th>
<th>̅𝑞</th>
</tr>
</thead>
<tbody>
<tr>
<td>𝑞</td>
<td>880</td>
<td>50</td>
<td>930</td>
</tr>
<tr>
<td>̅𝑞</td>
<td>50</td>
<td>20</td>
<td>70</td>
</tr>
<tr>
<td></td>
<td>930</td>
<td>70</td>
<td>1000</td>
</tr>
</tbody>
</table>

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</table>

\[ \text{lift} \left(\{𝑝\} \rightarrow \{𝑞\}\right) = 1.02 \]
even if they appear together in 88 % of the transactions

\[ \text{lift} \left(\{𝑟\} \rightarrow \{𝑠\}\right) = 4.08 \]
even if they rarely appear together.

In this case, using the confidence provides better results:

\[ \text{conf} \left(\{𝑝\} \rightarrow \{𝑞\}\right) = 94.6 \% \]
\[ \text{conf} \left(\{𝑟\} \rightarrow \{𝑠\}\right) = 28.6 \% \]
<table>
<thead>
<tr>
<th>#</th>
<th>Measure</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \phi )-coefficient</td>
<td>( \frac{P(A,B) - P(A)P(B)}{\sqrt{P(A)P(B)(1-P(A))(1-P(B))}} )</td>
</tr>
<tr>
<td>2</td>
<td>Goodman-Kruskal’s (( \lambda ))</td>
<td>( \frac{P(A,B)P(\bar{A},\bar{B})}{P(A,B)P(\bar{A},B) + \sum_j \max_k P(A_j,B_k) + \sum_i \max_j P(A_i,B_i) - \max_j P(A_j) - \max_k P(B_k) } )</td>
</tr>
<tr>
<td>3</td>
<td>Odds ratio (( \alpha ))</td>
<td>( \frac{P(A,B)P(\bar{A},B)}{P(A,B)P(\bar{A},\bar{B}) - \frac{P(A,B)P(\bar{A},B)P(\bar{A},\bar{B})}{\alpha}} = \frac{\alpha - 1}{\alpha + 1} )</td>
</tr>
<tr>
<td>4</td>
<td>Yule’s ( Q )</td>
<td>( \frac{\sqrt{P(A,B)P(\bar{A},\bar{B}) - \sqrt{P(A,B)P(\bar{A},B)}}}{\sqrt{P(A,B)P(\bar{A},\bar{B}) + \sqrt{P(A,B)P(\bar{A},B)}}} = \frac{\sqrt{\alpha - 1}}{\sqrt{\alpha + 1}} )</td>
</tr>
<tr>
<td>5</td>
<td>Yule’s ( Y )</td>
<td>( \frac{P(A,B)P(\bar{A},\bar{B}) - P(A,B)P(\bar{A},B)}{P(A,B)P(\bar{A},\bar{B}) + P(A,B)P(\bar{A},B) - P(A)P(\bar{B})} )</td>
</tr>
<tr>
<td>6</td>
<td>Kappa (( \kappa ))</td>
<td>( \frac{1 - P(A)P(\bar{B}) - P(A)P(\bar{B})}{P(A)P(B)} )</td>
</tr>
<tr>
<td>7</td>
<td>Mutual Information (( M ))</td>
<td>( \min(-\sum_i P(A_i)\log P(A_i) - \sum_j P(B_j)\log P(B_j)) )</td>
</tr>
<tr>
<td>8</td>
<td>J-Measure (( J ))</td>
<td>( \max(P(A,B)\log(P(B</td>
</tr>
<tr>
<td>9</td>
<td>Gini index (( G ))</td>
<td>( \max(P(A)[P(B</td>
</tr>
<tr>
<td>10</td>
<td>Support (( s ))</td>
<td>( \max(P(B</td>
</tr>
<tr>
<td>11</td>
<td>Confidence (( c ))</td>
<td>( \max\left( \frac{NP(A,B) + 1}{NP(A,B) + 1}, \frac{NP(\bar{A},\bar{B}) + 1}{NP(\bar{A},\bar{B}) + 1} \right) )</td>
</tr>
<tr>
<td>12</td>
<td>Laplace (( L ))</td>
<td>( \max\left( \frac{P(A)P(B)}{P(\bar{A}B)}, \frac{P(\bar{A})P(B)}{P(A\bar{B})} \right) )</td>
</tr>
<tr>
<td>13</td>
<td>Conviction (( V ))</td>
<td>( \max\left( \frac{P(A)P(B)}{P(\bar{A}B)}, \frac{P(\bar{A})P(B)}{P(A\bar{B})} \right) )</td>
</tr>
<tr>
<td>14</td>
<td>Interest (( I ))</td>
<td>( \frac{P(A,B)}{P(A)P(B)} )</td>
</tr>
<tr>
<td>15</td>
<td>cosine (( IS ))</td>
<td>( \sqrt{P(A)P(B)} )</td>
</tr>
<tr>
<td>16</td>
<td>Piatetsky-Shapiro’s (( PS ))</td>
<td>( P(A,B) - P(A)P(B) )</td>
</tr>
<tr>
<td>17</td>
<td>Certainty factor (( F ))</td>
<td>( \max\left( \frac{P(B</td>
</tr>
<tr>
<td>18</td>
<td>Added Value (( AV ))</td>
<td>( \max(P(B</td>
</tr>
<tr>
<td>19</td>
<td>Collective strength (( S ))</td>
<td>( \frac{P(A,B)P(\bar{A}\bar{B}) + P(\bar{A}B)P(\bar{B})}{P(A,B)} \times \frac{1 - P(A)P(B) - P(\bar{A})P(\bar{B})}{1 - P(A,B) - P(\bar{A}\bar{B})} )</td>
</tr>
<tr>
<td>20</td>
<td>Jaccard (( \zeta ))</td>
<td>( \frac{P(A,B) - P(A)P(B)}{P(A) + P(B) - P(A,B)} )</td>
</tr>
<tr>
<td>21</td>
<td>Klosgen (( K ))</td>
<td>( \sqrt{P(A,B)} \max(P(B</td>
</tr>
</tbody>
</table>

Many other measures... Many measures They have different properties. e.g. symetrical, anti-monotonic, etc.
References

Some content from these sources:

- Data Mining: The Textbook by Aggarwal (2015)
- Data Mining and Analysis Fundamental Concepts and Algorithms by Zaki & Meira (2014)