Maximal, Closed and Generator Itemsets

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Source code and datasets available in the SPMF library

Introduction

- In previous videos, we talked about **frequent itemset mining.**
- It is a data analysis task that has many applications.
- However, a problem is that the number of frequent itemsets is often very large.
- Besides, frequent itemsets often have some form of redundancy.
- Today, we will talk about this problem and a solution, which is to discover **concise representations of frequent itemsets.**

Frequent itemset mining

Input: A transaction database

Transaction	Items appearing in the transaction
T1	{pasta, lemon, bread, orange}
T2	{pasta, lemon}
T3	{pasta, orange, cake}
T4	{pasta, lemon, orange cake}

minsup = 2

Frequent itemset mining

Input: A transaction database

Transaction	Items appearing in the transaction
T1	{pasta, lemon, bread, orange}
T2	{pasta, lemon}
T3	{pasta, orange, cake}
T4	{pasta, lemon, orange cake}

minsup = 2

Output: The frequent itemsets
{lemon}, {pasta}, {orange}, {cake}, {lemon, pasta}, {lemon, orange}, {pasta, orange}, {pasta, cake}, {orange, cake}, {lemon, pasta, orange}, {pasta, orange, cake}

Frequent itemsets

{pasta}	support = 4
{lemon}	support = 3
{orange}	support = 3
{cake}	support = 2
{pasta, lemon}	support: 3
{pasta, orange}	support: 3
{pasta, cake}	support: 2
{lemon, orange}	support: 2
{orange, cake}	support: 2
{pasta, lemon, orange}	support: 2
{pasta, orange, cake}	support: 2

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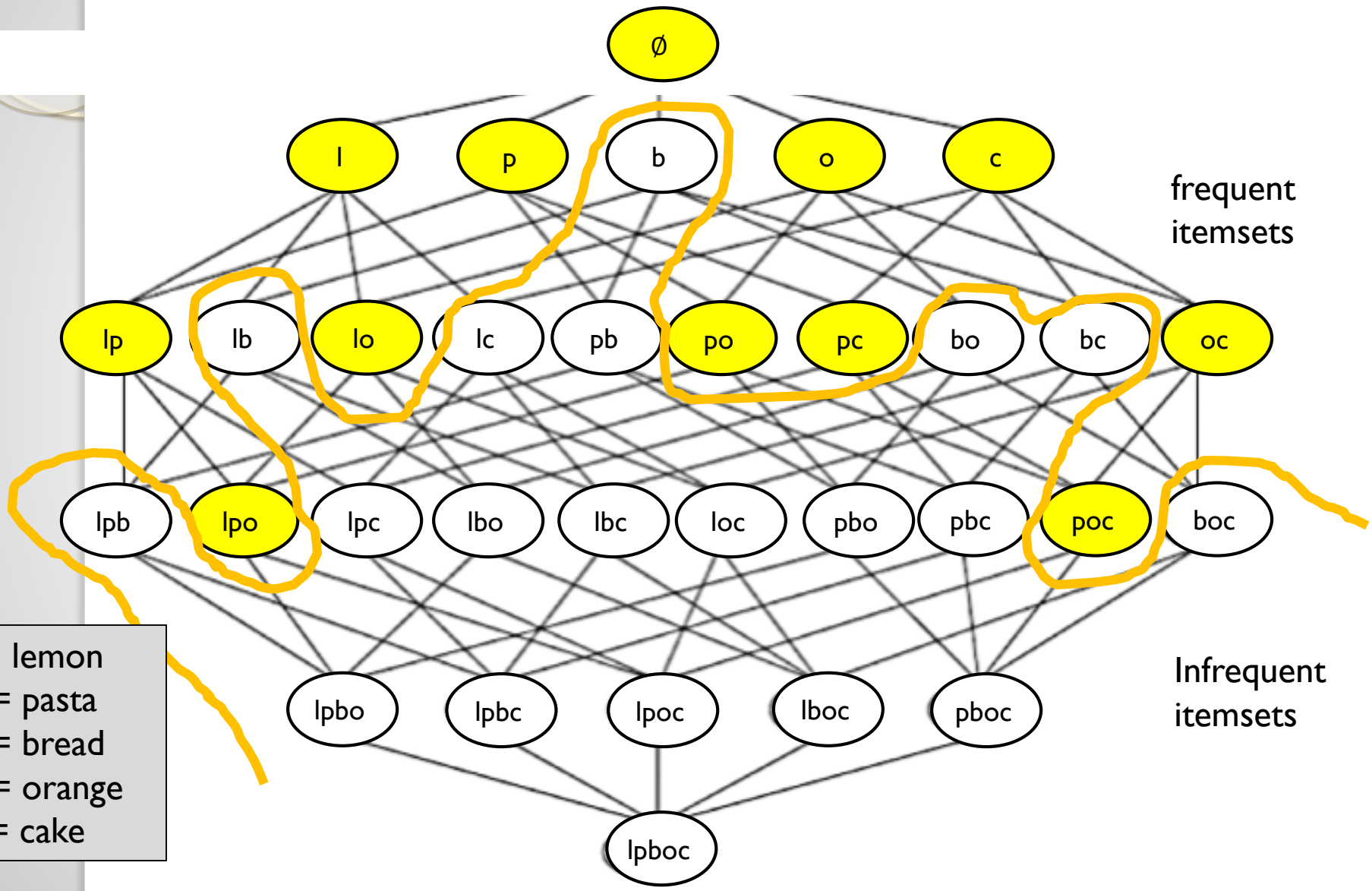
{lemon, orange}	support: 2
{orange, cake}	support: 2
{pasta, lemon, orange}	support: 2
{pasta, orange, cake}	support: 2

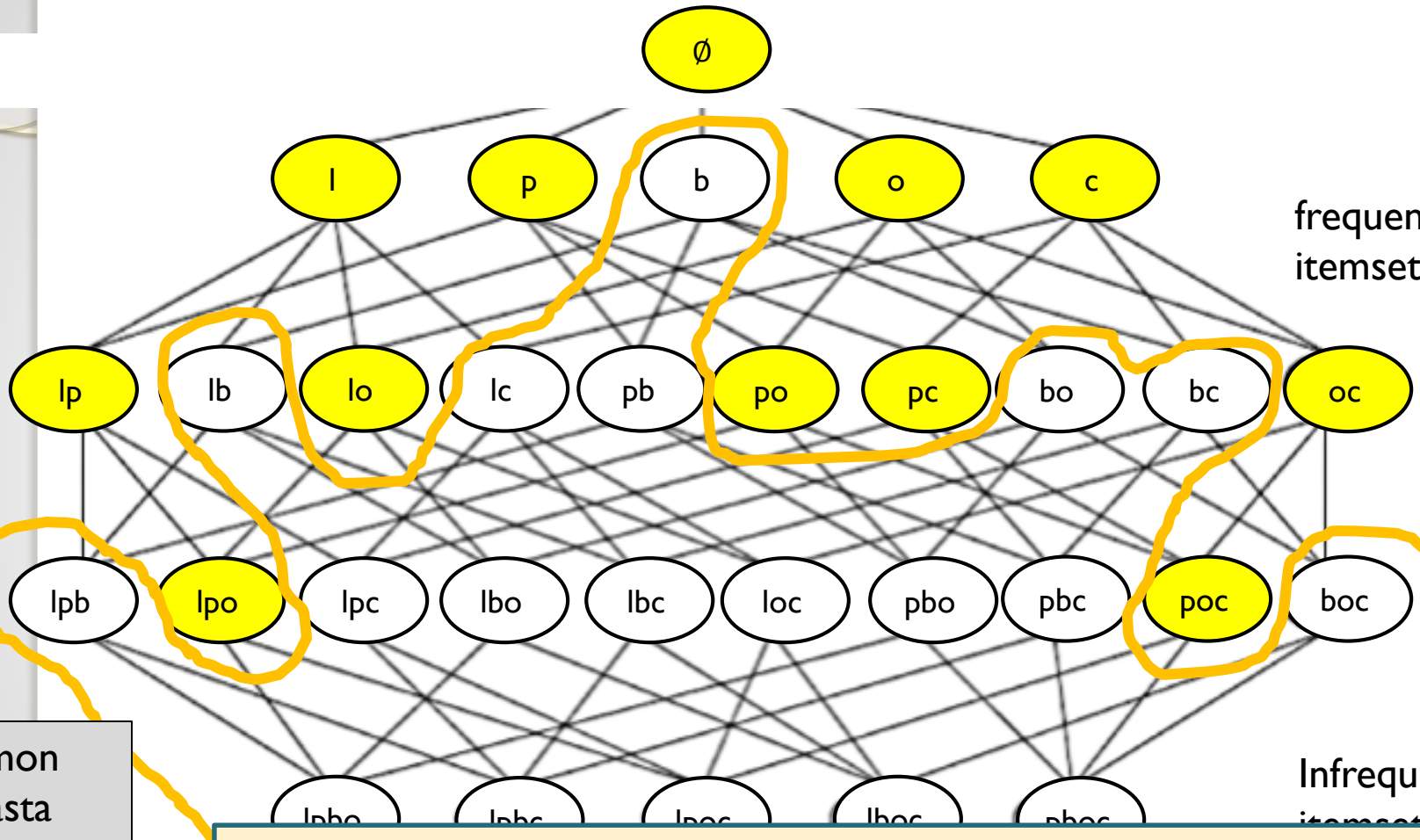
There are 10 frequent itemsets!

minsup =2 The frequent itemsets



l = lemon
p = pasta
b = bread
0 = orange
c = cake

minsup =2 The frequent itemsets



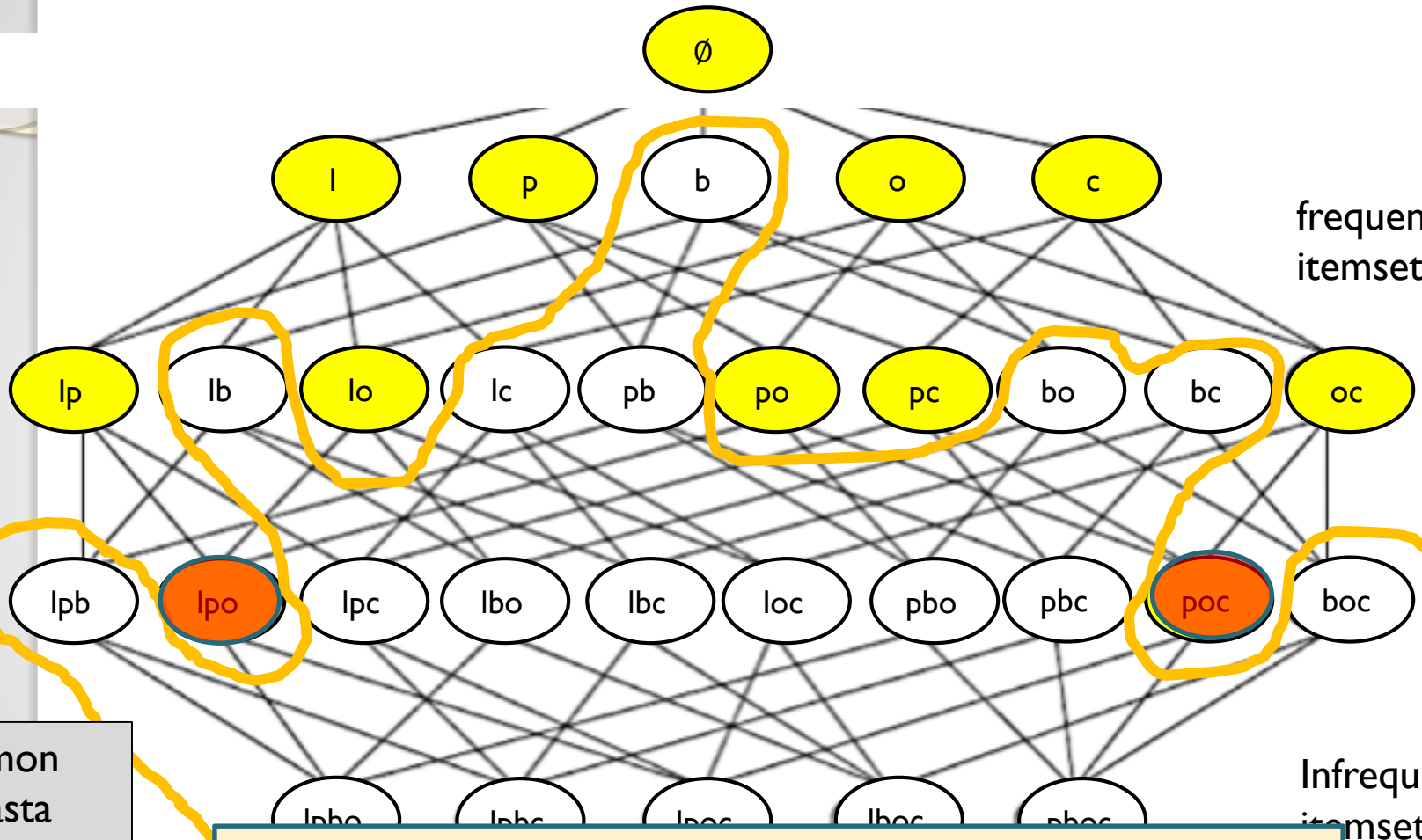
frequent itemsets

Infrequent itemsets

l = lemon
p = pasta
b = bread
0 = orange
c = cake

We can observe that if an itemset is frequent then all its subsets are frequent

minsup =2 The frequent itemsets



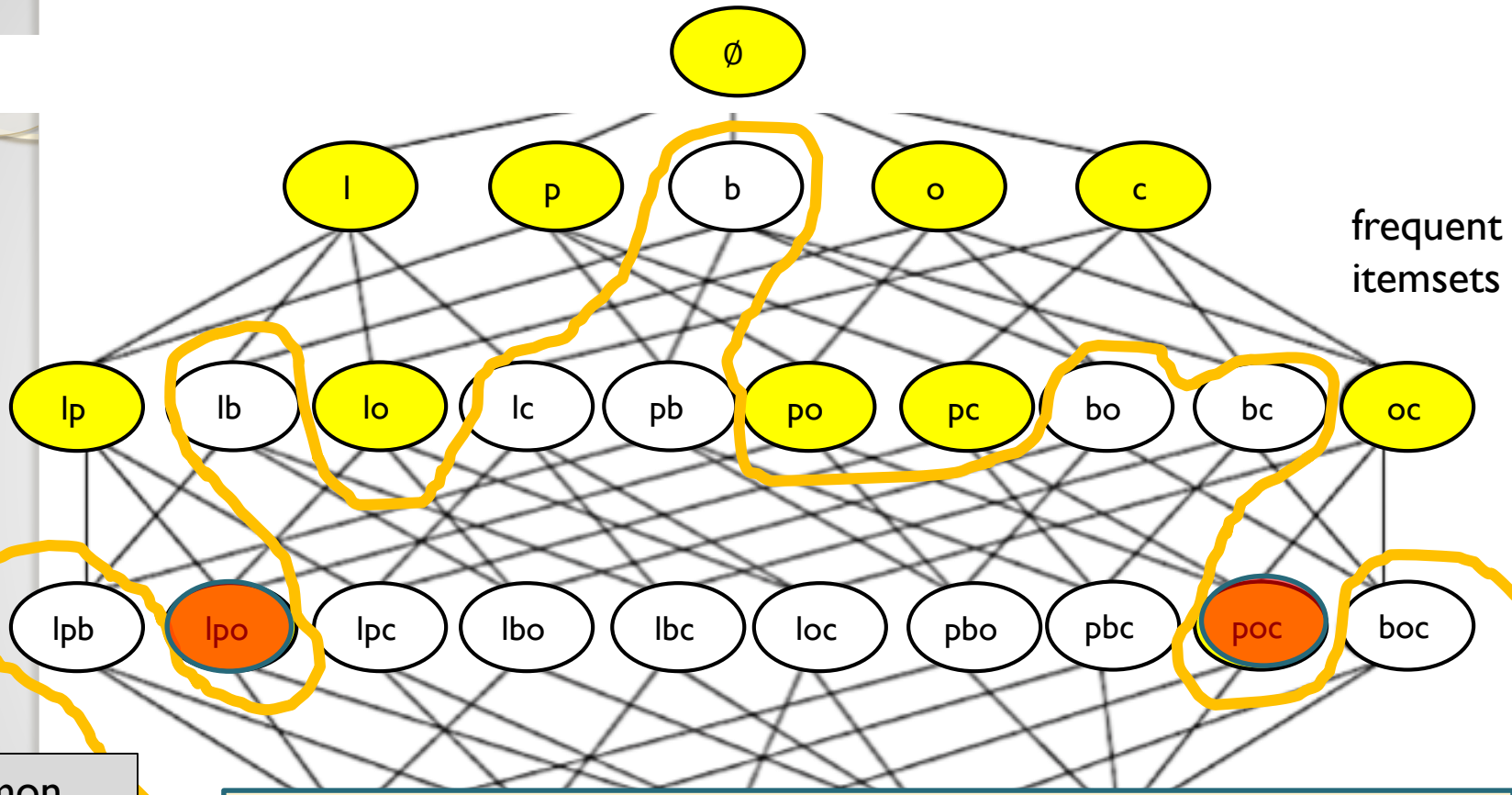
frequent itemsets

Infrequent itemsets

l = lemon
p = pasta
b = bread
0 = orange
c = cake

We call the *largest* frequent itemsets the ***maximal frequent itemsets***

minsup =2 The frequent itemsets



frequent itemsets

l = lemon
p = pasta
b = bread
0 = orange
c = cake

Maximal frequent itemsets are interesting because if we find them, we can recover all the other frequent itemsets.

Recovering all frequent itemsets

Definition: A frequent **itemset** X is **maximal** if there is no frequent itemset Y such that $X \subset Y$.

In this example *(minsup = 2)*, there are only two maximal itemsets:

{pasta, lemon, orange}	support: 2
{pasta, orange, cake}	support: 2

Maximal frequent itemsets

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We can **recover** all other frequent itemsets from the maximal itemsets by simply enumerating their subsets:

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{lemon}, {pasta}, {orange}, {cake}, {lemon, pasta}, {lemon, orange}, {pasta, orange}, {pasta, cake}, {orange, cake},

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We can **recover** all other frequent itemsets from the maximal itemsets by simply enumerating their subsets:

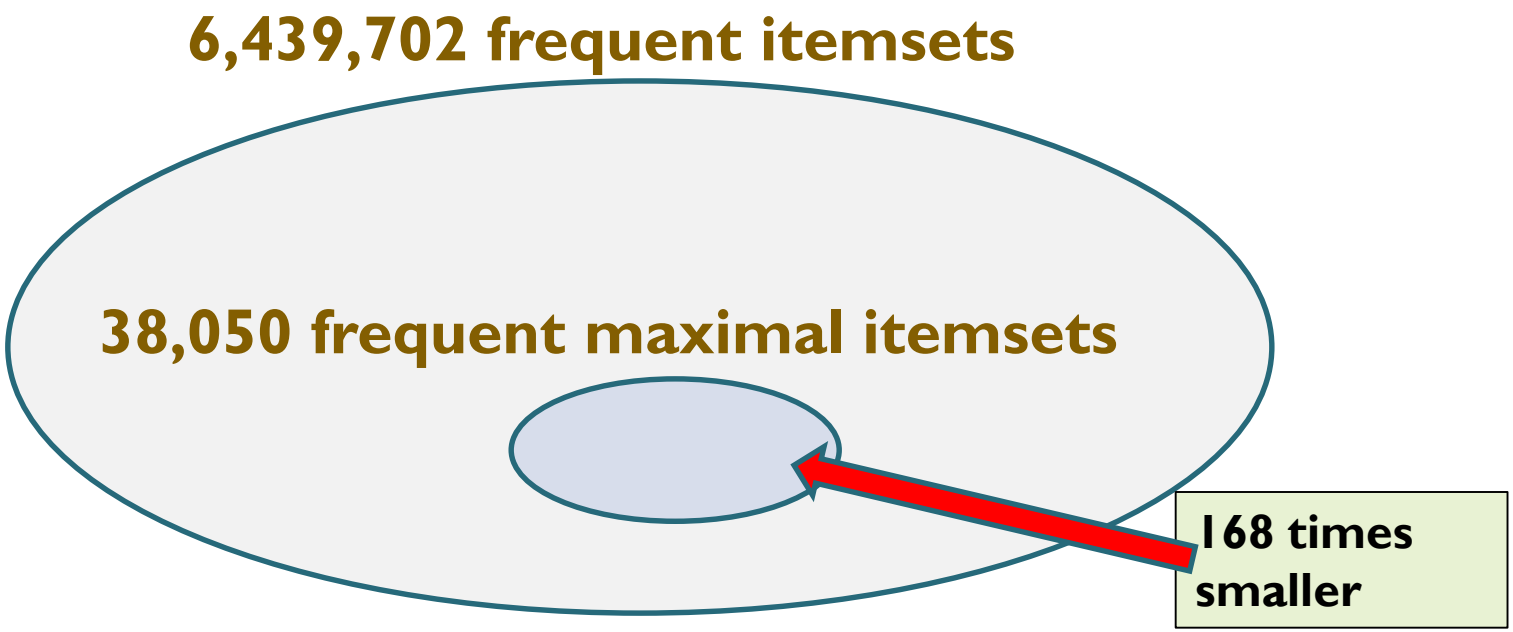
{lemon}, {pasta}, {orange}, {cake}, {lemon, pasta}, {lemon, orange}, {pasta, orange}, {pasta, cake}, {orange, cake},

But we cannot recover their support!

Maximal itemsets are compact

If we find the maximal frequent itemsets, we can find **much less** itemsets.

Example: ***Chess*** dataset and minsup =0.4



How to find the maximal itemsets?

- **Naïve approach**
  - by post-processing
  - **inefficient!**
- **Efficient algorithms:**
  - GenMax: a modified version of ECLAT
  - FPMax: a modified version of FP-Growth
  - LCMMax: a modified version of LCM
  - … and many others!

- Those algorithms adopt various strategies to find the maximal itemsets without having to find all frequent itemsets.

What are the alternatives?

- Is there some other ways of summarizing the frequent itemsets?
- Another popular representation of frequent itemsets is the **closed itemsets** (Pasquier et al., 1999)

Could we do better?

- Is there some other ways of summarizing the frequent itemsets?
- Another popular representation of frequent itemsets is the **closed itemsets** (Pasquier et al., 1999)

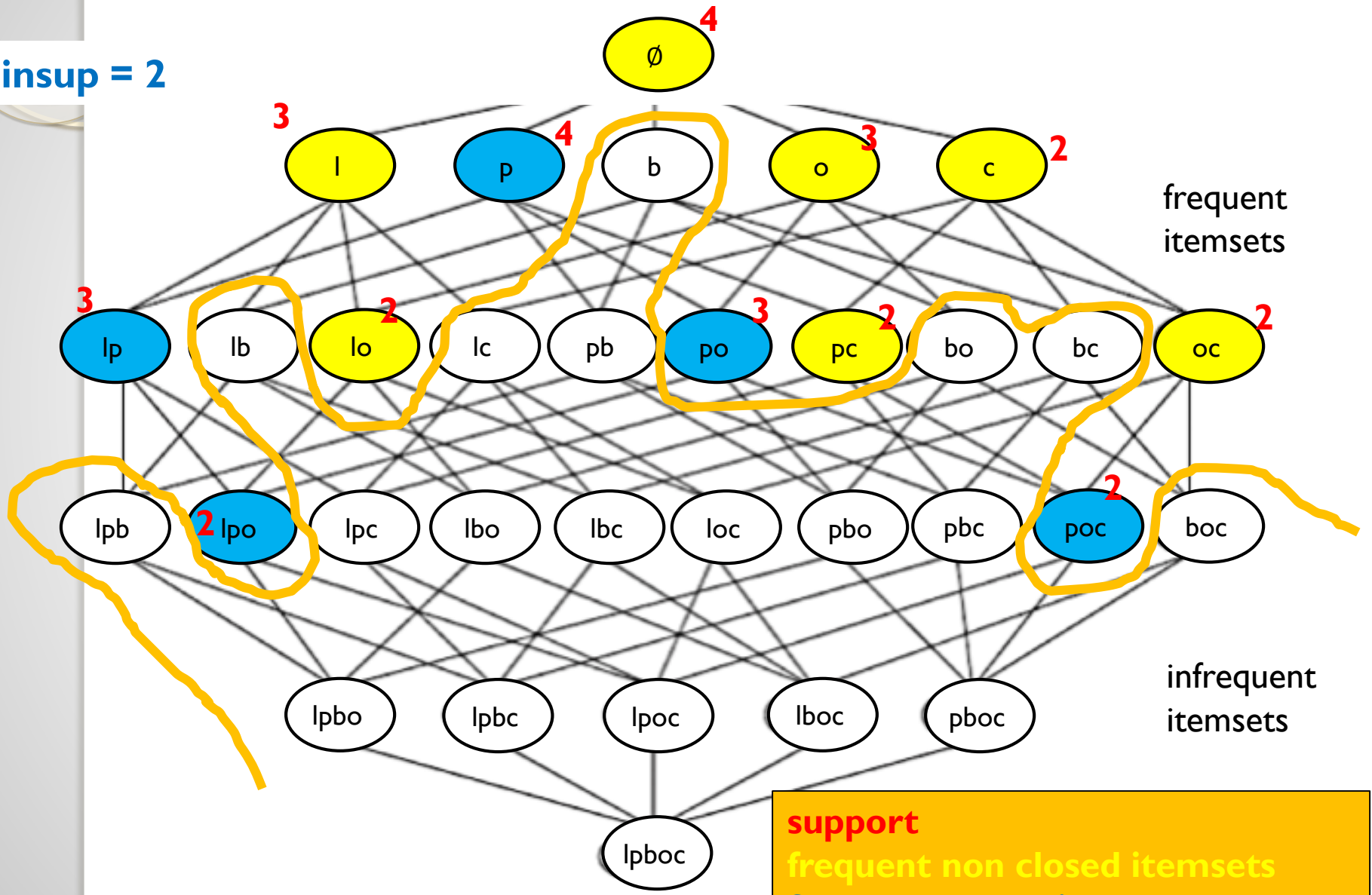
Definition: A (frequent) **itemset** X is **closed** if there exists no (frequent) itemset Y such that $X \subset Y$ and $\sup(X) = \sup(Y)$.

Closed itemsets

{pasta}	**support = 4**	**closed**
{lemon}	support = 3	
{orange}	support = 3	
{cake}	support = 2	
{pasta, lemon}	**support: 3**	**closed**
{pasta, orange}	**support: 3**	**closed**
{pasta, cake}	support: 2	
{lemon, orange}	support: 2	
{orange, cake}	support: 2	
{pasta, lemon, orange}	**support: 2**	**closed**
{pasta, orange, cake}	**support: 2**	**closed**

minsup =2 The frequent closed itemsets

minsup = 2



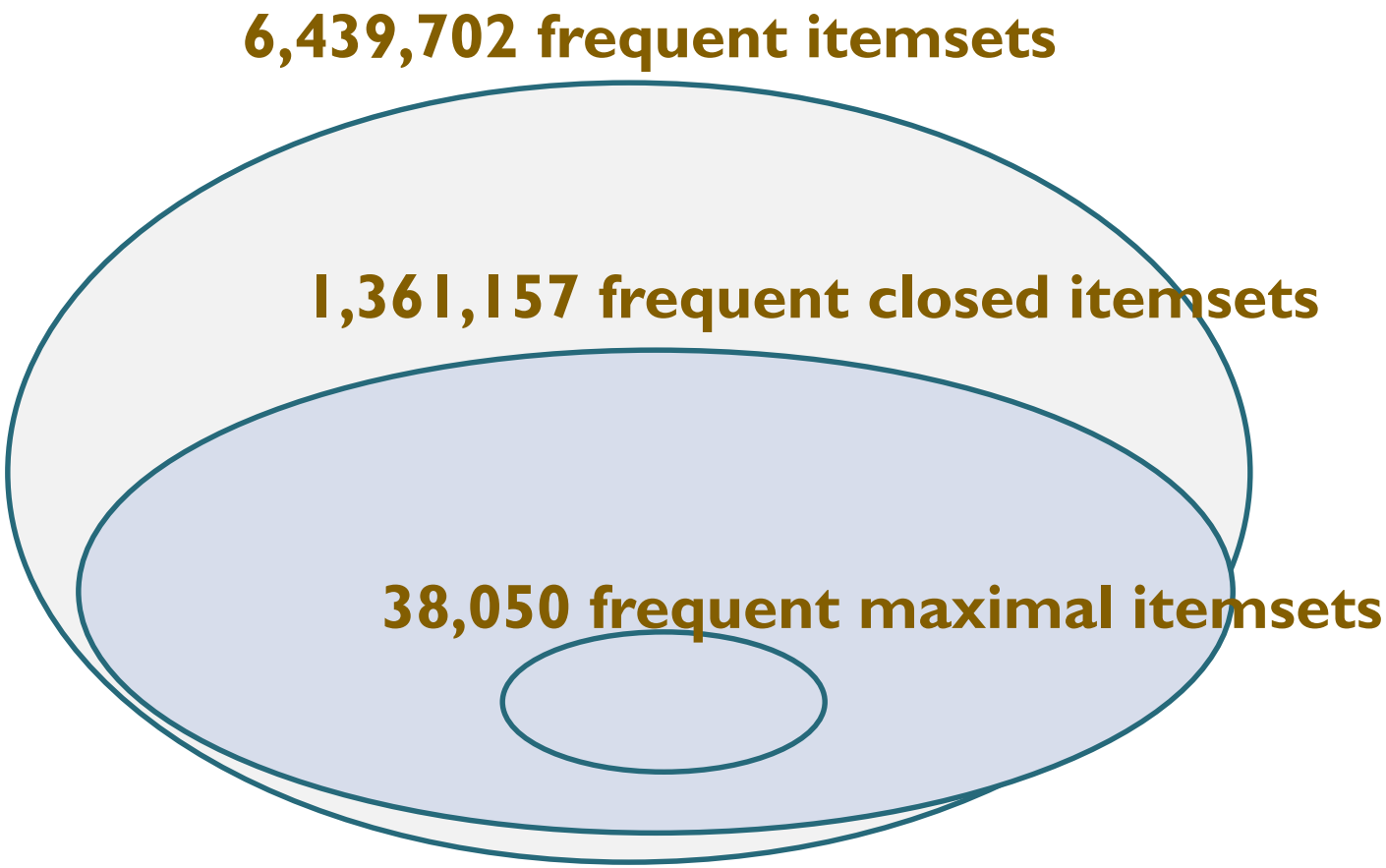
support

frequent non closed itemsets

frequent closed itemsets

Closed itemsets are compact

Example: ***Chess*** dataset and minsup =0.4



Recovering all frequent itemsets

Definition: A (frequent) **itemset** X is **closed** if there exists no (frequent) itemset Y such that $X \subset Y$ and $\sup(X) = \sup(Y)$.

In this example *(minsup = 2)*, there are only five closed itemsets:

{pasta}	**support = 4**	**closed**
{pasta, lemon}	**support: 3**	**closed**
{pasta, orange}	**support: 3**	**closed**
{pasta, lemon, orange}	**support: 2**	**closed**
{pasta, orange, cake}	**support: 2**	**closed**

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{pasta, orange}	**support: 3**	**closed**
{pasta, lemon, orange}	**support: 2**	**closed**
{pasta, orange, cake}	**support: 2**	**closed**

By enumerating all their subsets, we can find all the other frequent itemsets:

{pasta, cake}, {lemon, orange}, {orange, cake},
{lemon,{orange} ,{cake}

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{pasta, lemon, orange}	**support: 2**	**closed**
{pasta, orange, cake}	**support: 2**	**closed**

By enumerating all their subsets, we can find all the other frequent itemsets:

{pasta, cake}, {lemon, orange}, {orange, cake}, {lemon,{orange} ,{cake}

But how to find the support of these itemsets? →

Steps to find the support of a frequent itemset X from closed itemsets

1. Find the closure of X.
2. Return the support of the closure of X

Definition: The closure of an **itemset** X is the smallest closed itemset Y such that $X \subseteq Y$. It is denoted as c(X)

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{pasta, orange, cake}	**support: 2**	**closed**

The closure of {cake} is c({cake}) ???

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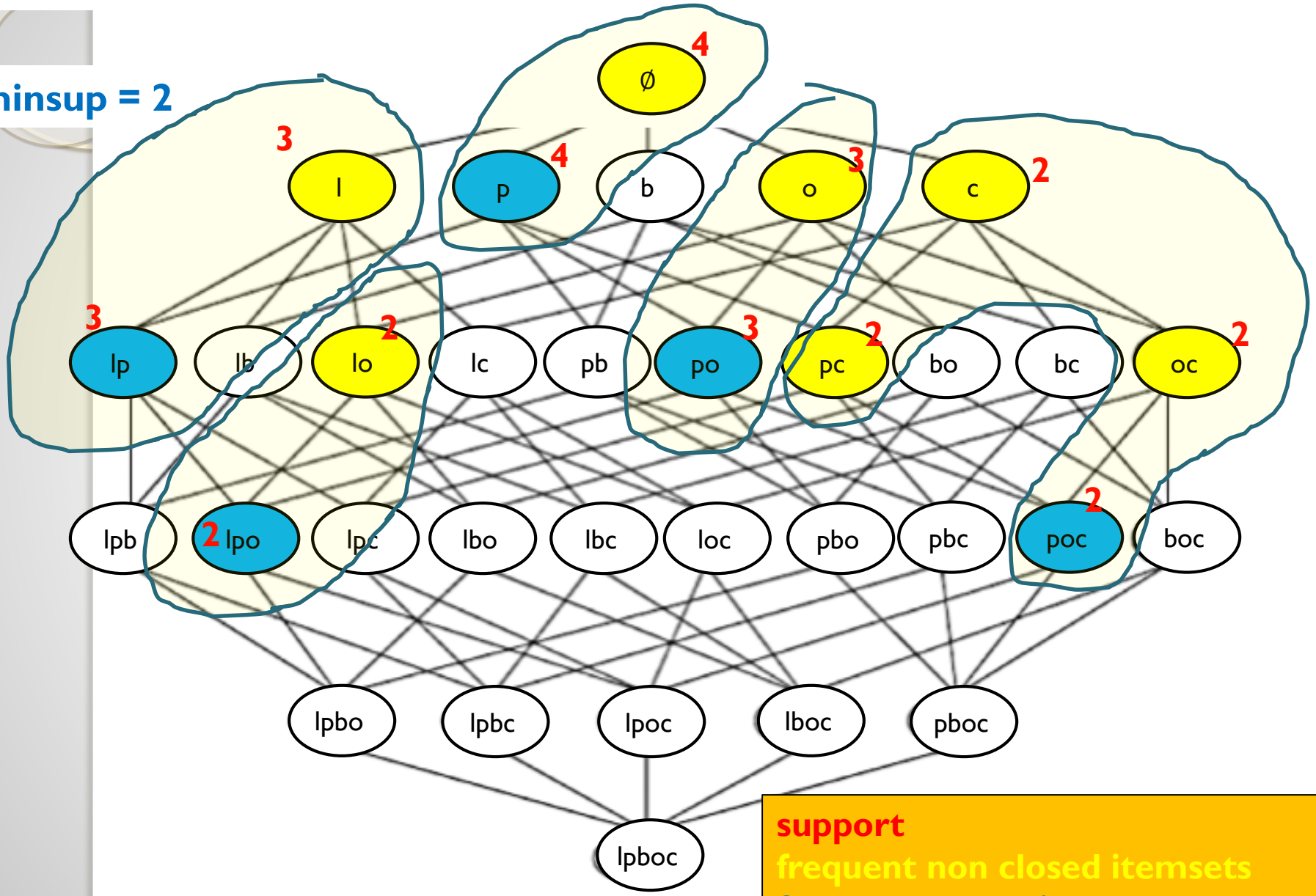
{pasta, orange}	**support: 3**	**closed**
{pasta, lemon, orange}	**support: 2**	**closed**
{pasta, orange, cake}	**support: 2**	**closed**

The closure of {cake} is c({cake}) = {pasta,orange,cake}

Thus, the support of {cake} is the same, that is 2.

Itemsets and their closures

minsup = 2



support

frequent non closed itemsets

frequent closed itemsets

Itemsets that have the same closure

- They all have the same support
- They all appear in the same transactions.

Example:

{cake}
{pasta, cake},
{orange, cake},
{pasta, orange, cake}

They all have a support of 2
They all appears in T3 and T4

Trans.	Items
T1	{pasta, lemon, bread, orange}
T2	{pasta, lemon}
T3	{pasta, orange, cake}
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Example:

{cake}
{pasta, cake},
{orange, cake},
{pasta, orange, cake}

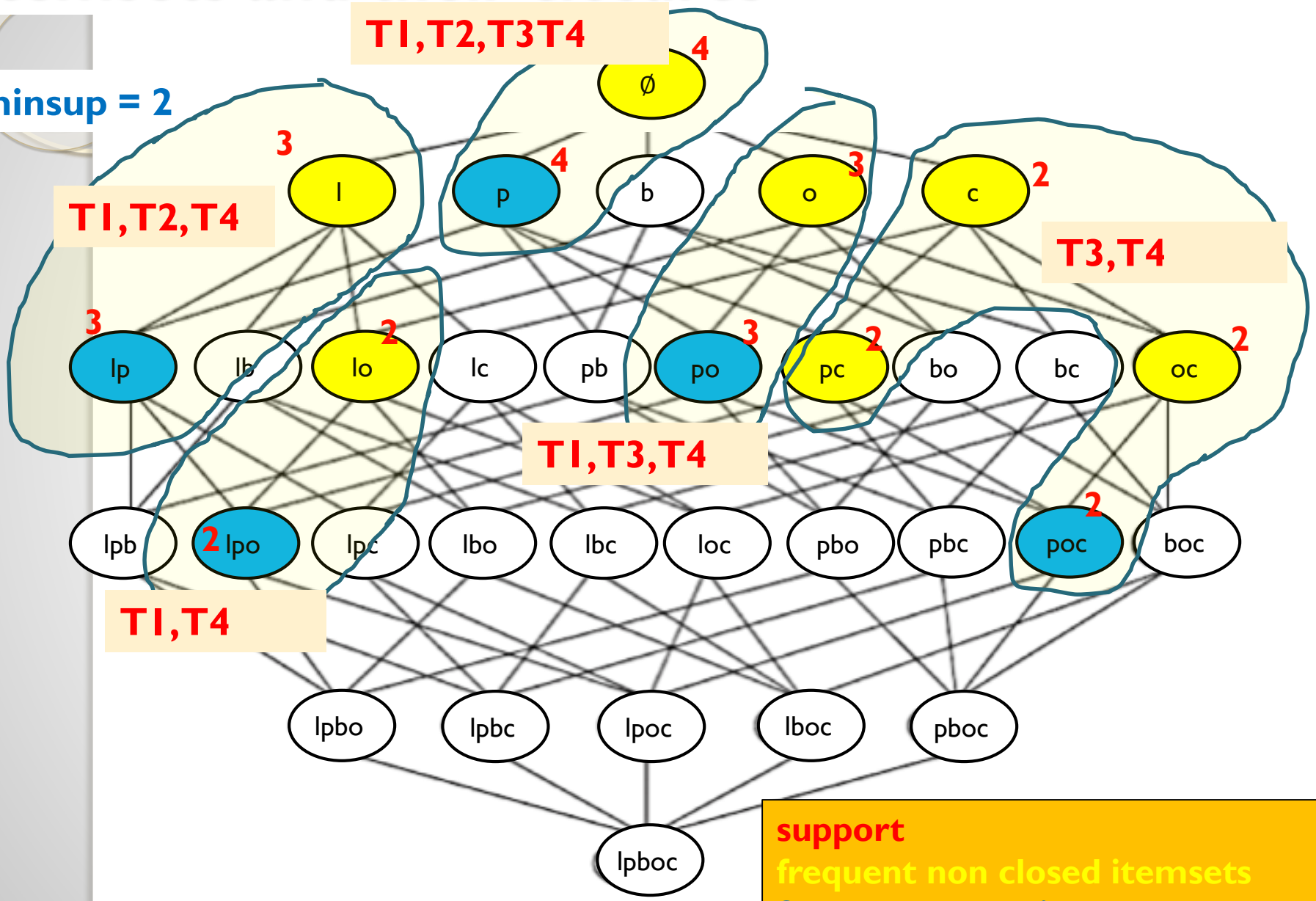
They all have a support of 2
They all appears in T3 and T4

Trans.	Items
T1	{pasta, lemon, bread, orange}
T2	{pasta, lemon}
T3	{pasta, orange, cake}
T4	{pasta, lemon, orange cake}

Observation:
The closure is the intersection of transactions T3 and T4

Itemsets and their closures

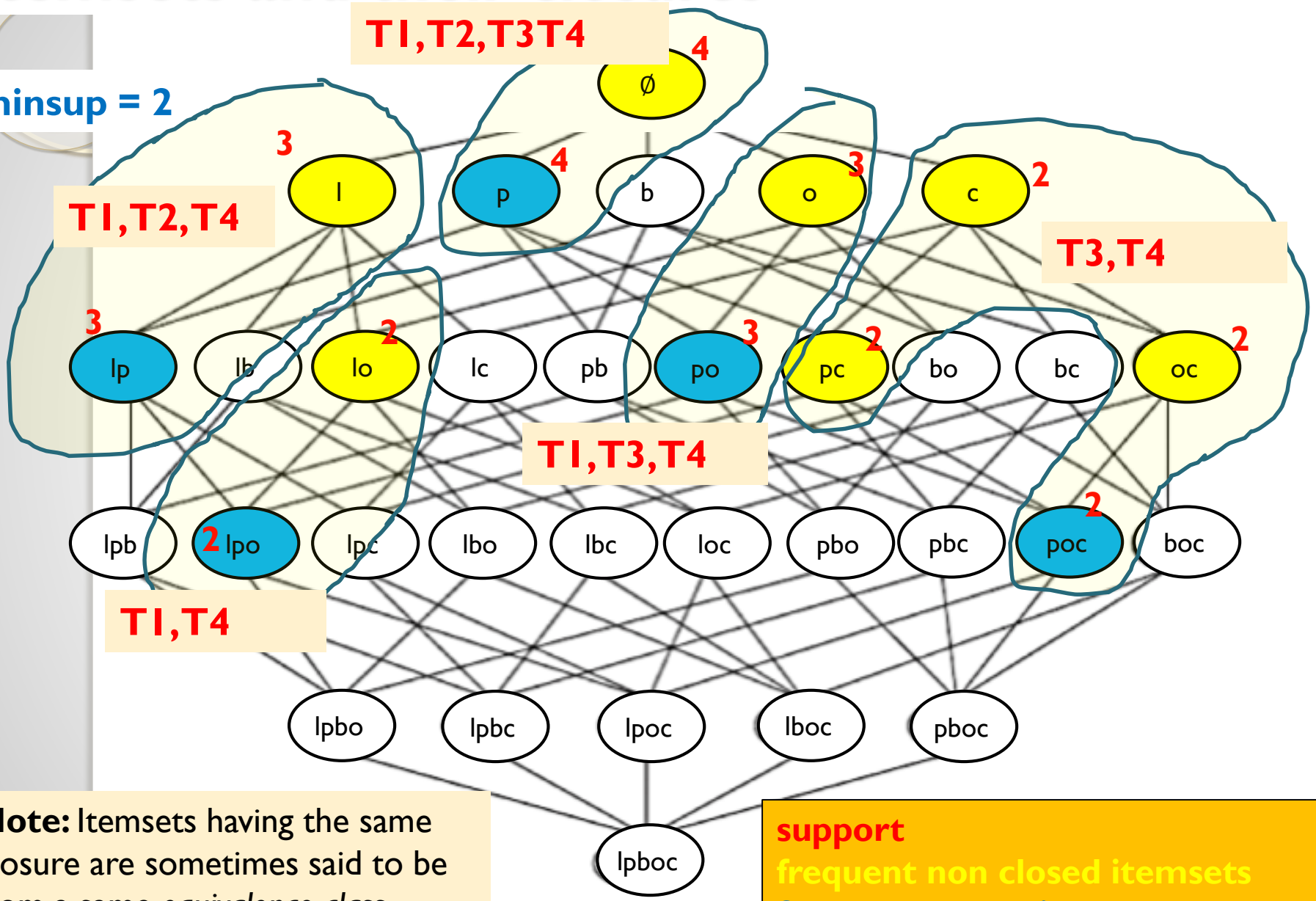
minsup = 2



support
frequent non closed itemsets
frequent closed itemsets

Itemsets and their closures

minsup = 2



Note: Itemsets having the same closure are sometimes said to be from a same *equivalence class*.

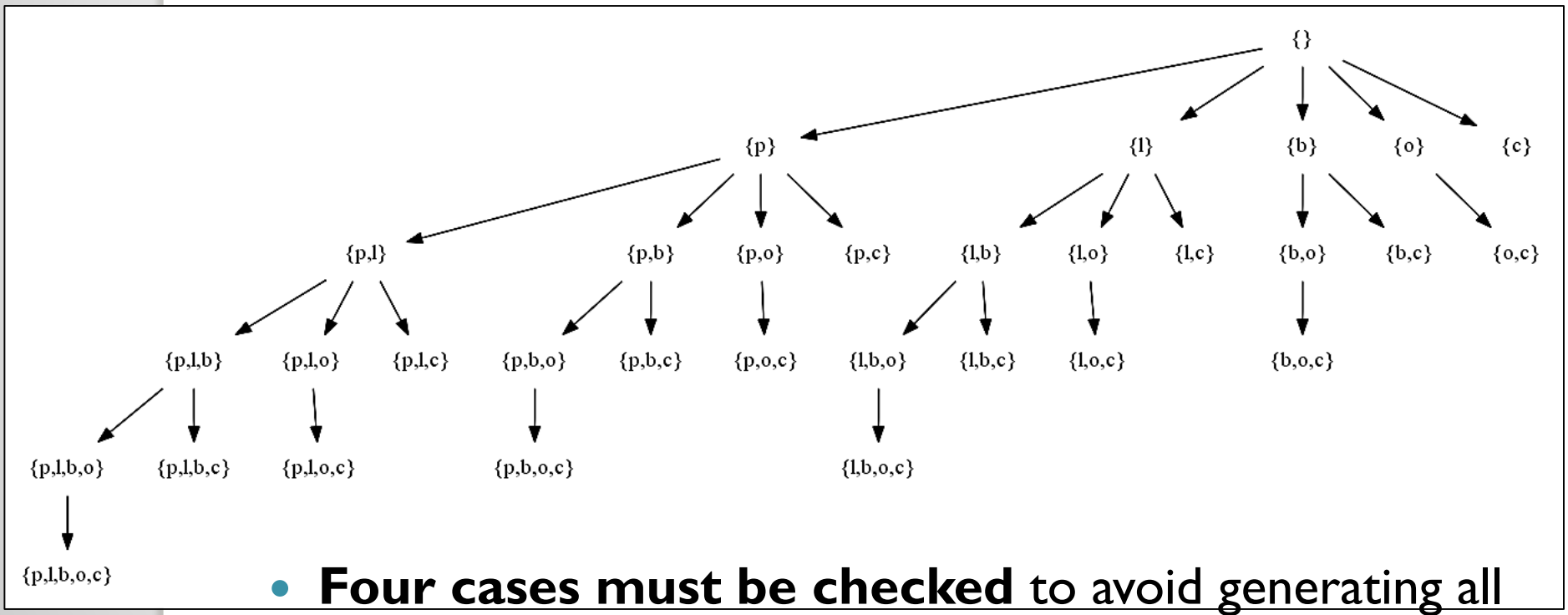
support
frequent non closed itemsets
frequent closed itemsets

How to find the closed itemsets?

- **Naïve approach**:
  - by post-processing
  - **inefficient!**
- **Efficient algorithms:**
  - Charm, DCI_Closed, FPClose, Closet, LCM, etc.
  - Find frequent itemsets by avoiding generating all frequent itemsets.
  - Several strategies and properties.

Charm (Zaki, 2001)

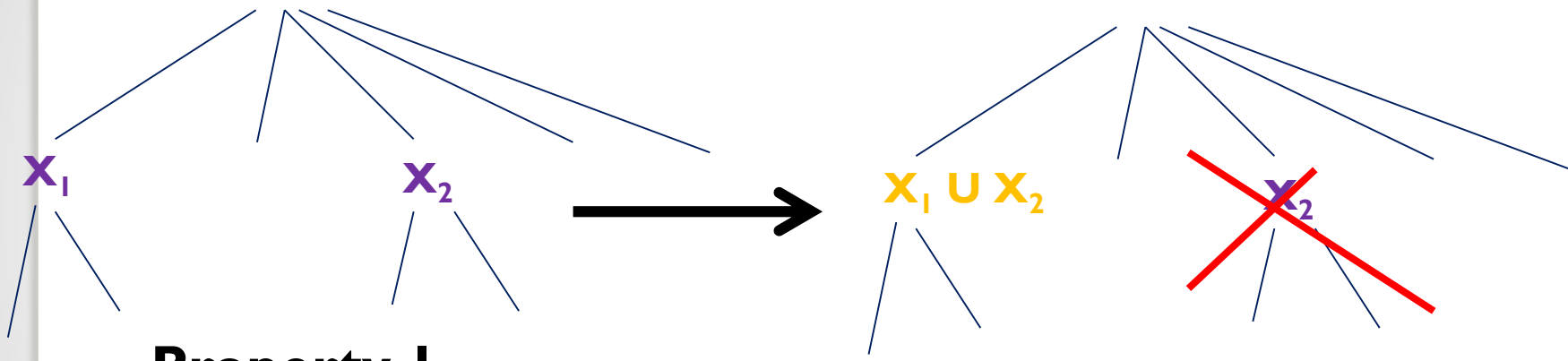
- Based on ECLAT
- Depth-first search



- **Four cases must be checked** to avoid generating all frequent itemsets→

Four cases

Suppose that the algorithm is considering combining an itemset X_1 with an itemset X_2 such that $X_2 \succ X_1$.



Property 1

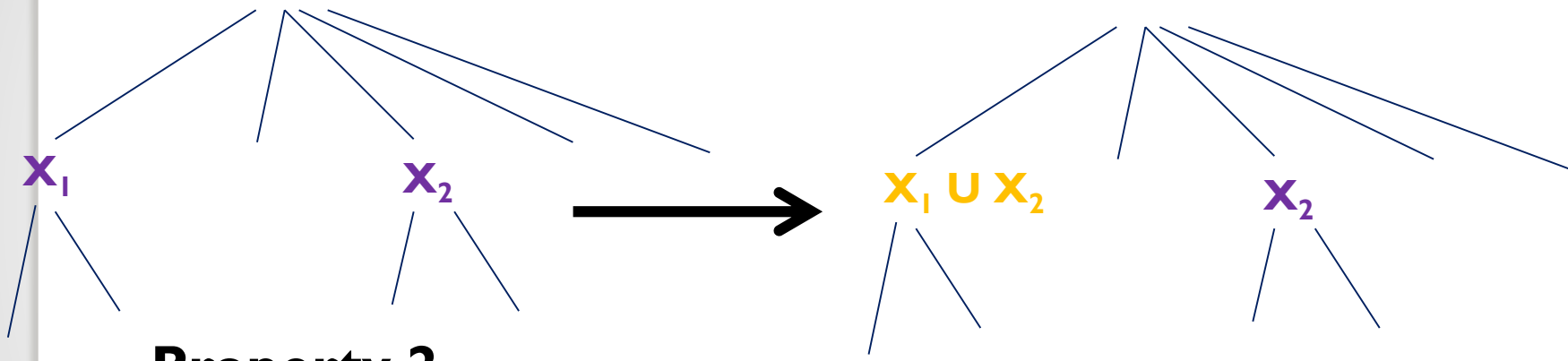
IF $t(X_1) = t(X_2)$, **THEN** $t(X_1) = t(X_2) = t(X_1 \cap X_2)$.

- Thus, we can replace all occurrences of X_1 by $X_1 \cup X_2$.
- We do not need to consider X_2.

Notation: t(X) denotes the set of transactions containing an itemset X

Four cases

Suppose that the algorithm is considering combining an itemset X_1 with an itemset X_2 such that $X_2 \succ X_1$.



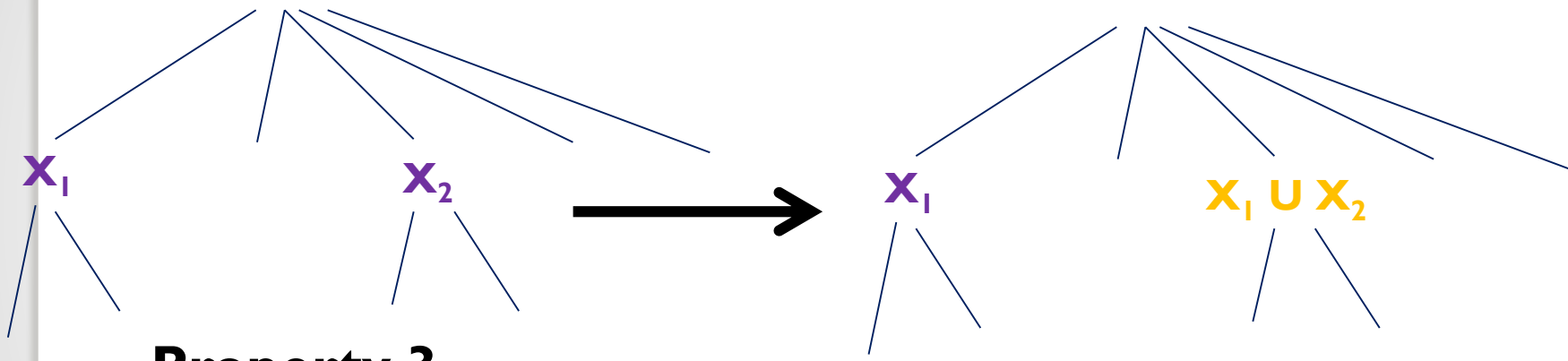
Property 2

IF $t(X_1) \subset t(X_2)$, **THEN** $t(X_1) = t(X_1 \cap X_2) \neq t(X_2)$.

- Thus, we can replace all occurrences of X_1 by $X_1 \cup X_2$.
- But we must keep X_2.

Four cases

Suppose that the algorithm is considering combining an itemset X_1 with an itemset X_2 such that $X_2 \succ X_1$.



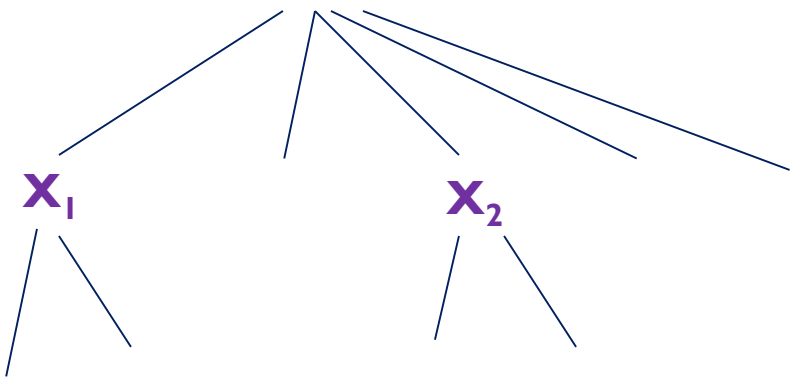
Property 3

Si $t(X_1) \supset t(X_2)$, **alors** $t(X_2) = t(X_1 \cap X_2) \neq t(X_1)$.

- Thus, we can replace all occurrences of X_2 by $X_1 \cup X_2$.
- But we must keep X_1.

Four cases

Suppose that the algorithm is considering combining an itemset X_1 with an itemset X_2 such that $X_2 \succ X_1$.



Property 4

IF $t(X_1) \neq t(X_2)$, **then** $t(X_2) \neq t(X_1 \cap X_2) \neq t(X_1)$.

- We cannot remove anything.

Charm

- These four properties allows to
  - eliminate several frequent itemsets
  - thus, reduce the search space
- However, the algorithm can still generate several itemsets that are non closed.
- We must add a **closure checking test**.

Closure checking

- For each **frequent itemset X found**, we compare it to the other **frequent closed itemsets** found until now.
- If **X** is contained in an itemset that has been already found, we do not keep **X.**
- Otherwise, **X** is closed. It is added to the set of **closed itemsets**.
- How to implement this test?
  - store closed itemsets in a hash table
  - hash function is the list of transaction ids t(.).

Pseudocode of Charm

```
CHARM (δ ⊆ I × T, minsup):
1.  Nodes = {I_j × t(I_j) : I_j ∈ I ∧ |t(I_j)| ≥ minsup}
2.  CHARM-EXTEND (Nodes, C)

CHARM-EXTEND (Nodes, C):
3.  for each X_i × t(X_i) in Nodes
4.      NewN = ∅ and X = X_i
5.      for each X_j × t(X_j) in Nodes, with j ≻ i
6.          X = X ∪ X_j and Y = t(X_i) ∩ t(X_j)
7.          CHARM-PROPERTY(Nodes, NewN)
8.      if NewN ≠ ∅ then CHARM-EXTEND (NewN)
9.      C = C ∪ X //
```

vérifier le test de closure

```
CHARM-PROPERTY (Nodes, NewN):
10. if (|Y| ≥ minsup) then
11.     if t(X_i) = t(X_j) then //Property 1
12.         Remove X_j from Nodes
13.         Replace all X_i with X
14.     else if t(X_i) ⊂ t(X_j) then //Property 2
15.         Replace all X_i with X
16.     else if t(X_i) ⊃ t(X_j) then //Property 3
17.         Remove X_j from Nodes
18.         Add X × Y to NewN
19.     else if t(X_i) ≠ t(X_j) then //Property 4
20.         Add X × Y to NewN
```

Nodes the set of frequent itemsets of size 1

I *is the set of items* ***T*** *is the set of transactions*

C *is the set of all closed itemsets found*

X x t(X) *denotes an* ***X*** *and its list of transaction* ***t(X)***

Optimizations

- Implementation with diffsets (**dCharm**).
- Implementation with bit vectors.
- Use a **triangular matrix** to count the support of itemsets of size 2 by scanning the database once.
- …

	E	D	C	B
A	0	0	0	0
B	0	0	0	
C	0	0		
D	0			

illustration: szathmary, 2006

Maximal vs Closed Itemsets

- **Closed frequent itemsets**
  - Can recover all frequent itemsets
  - Can recover the support
- **Maximal frequent itemsets**
  - Can recover all frequent itemsets
  - Cannot recover the support
  - A subset of closed itemsets, sometimes much smaller.

Is there other compact representations of frequent itemsets? Yes →

The generator itemsets (or key itemsets)

Definition: A (frequent) **itemset** X is a **generator** if there exists no (frequent) itemset Y such that $Y \subset X$ and $\sup(X) = \sup(Y)$.

The generator itemsets (or key itemsets)

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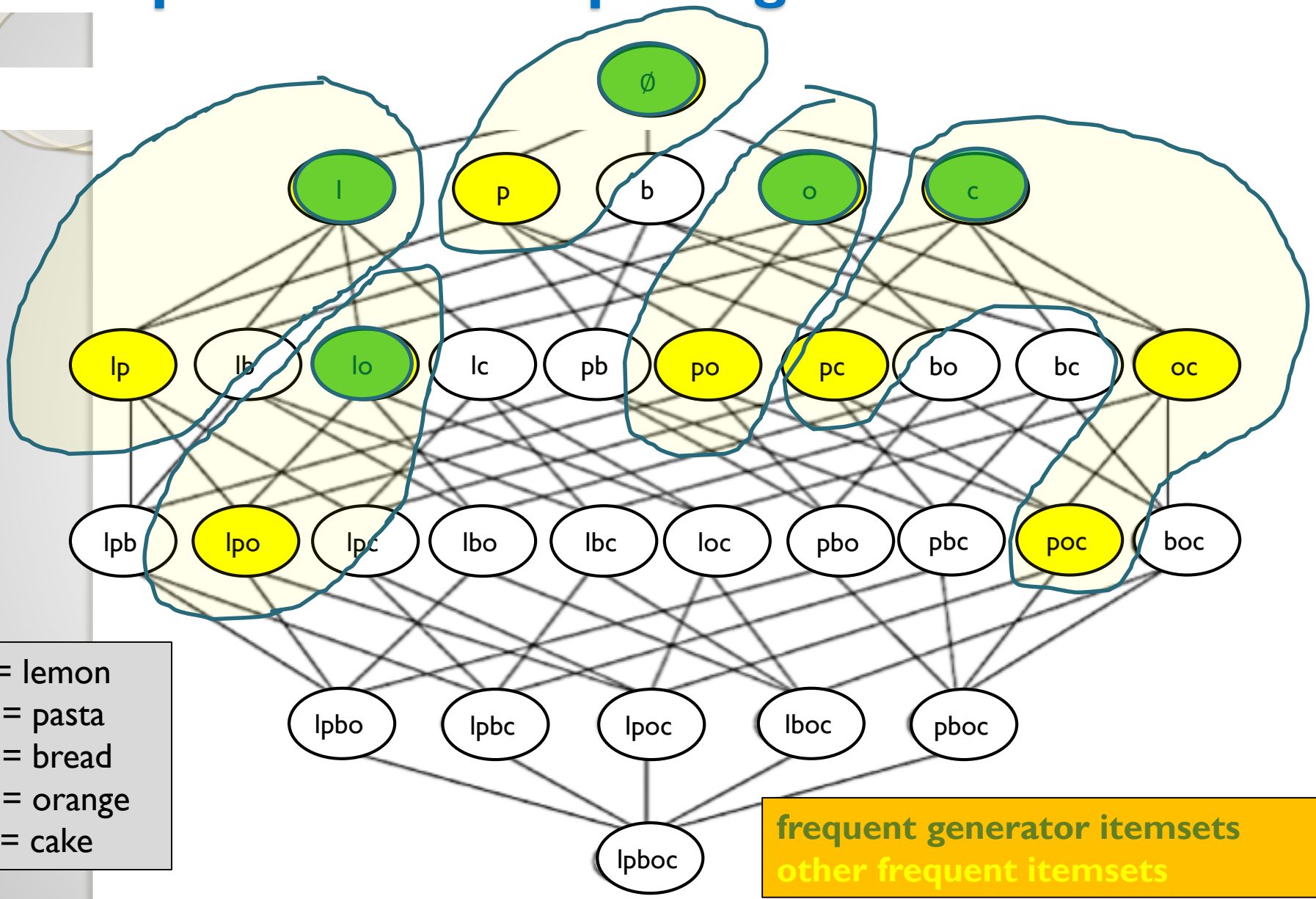
In the example *(minsup = 2)*, there are five **frequent generator itemsets**:

{}	support: 4
{lemon}	support: 3
{orange}	support: 3
{cake}	support: 2
{lemon,orange}	support: 2

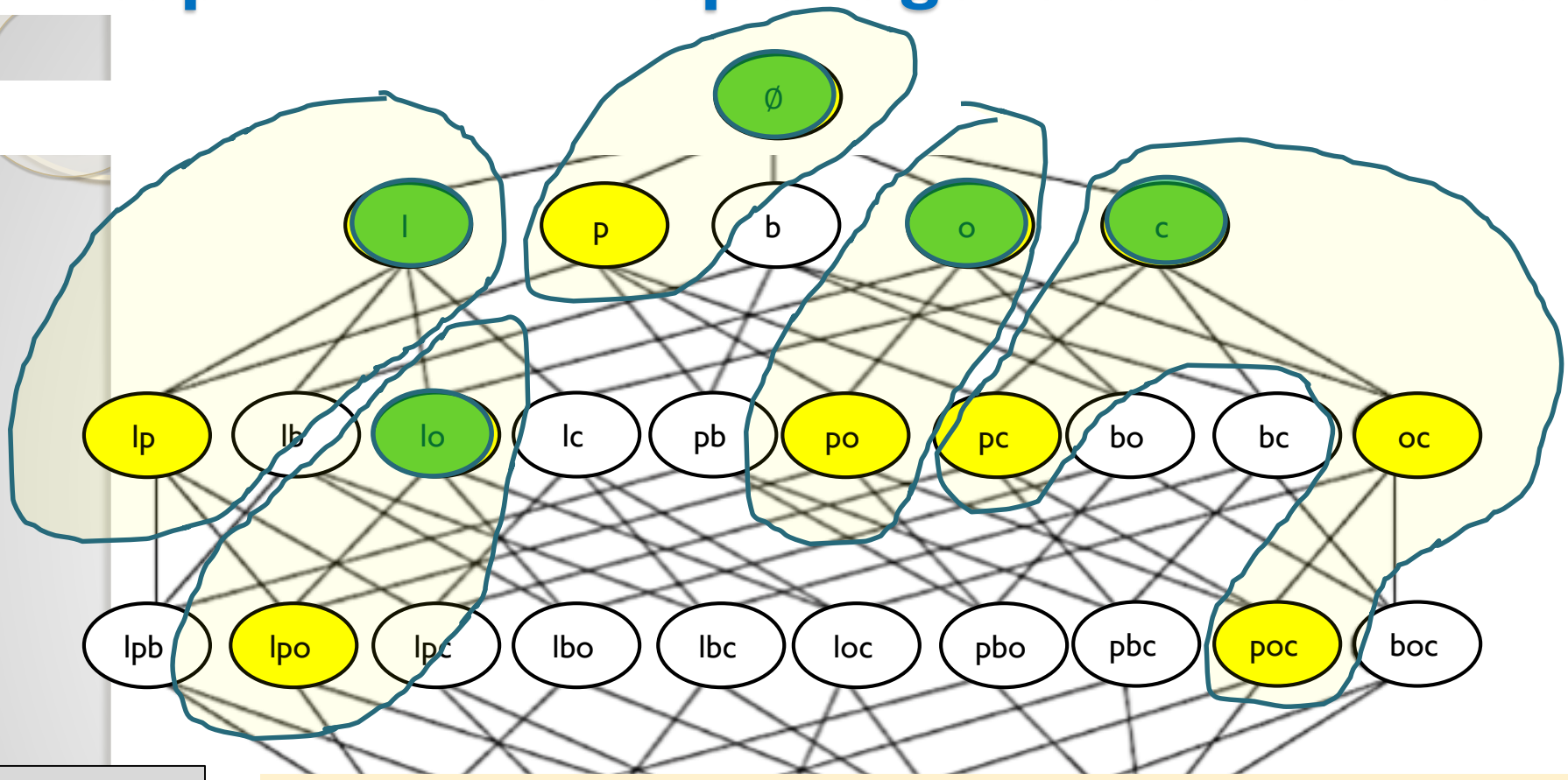
minsup =2 The frequent generator itemsets



l = lemon
p = pasta
b = bread
0 = orange
c = cake

frequent generator itemsets
other frequent itemsets

minsup =2 The frequent generator itemsets



l = lemon
p = pasta
b = bread
0 = orange
c = cake

Some observations:

- The **empty set** can be a generator
- While each equivalence class has always one closed itemset, it could have more than one generator (not in this example)
- In some cases, a closed itemset can be a generator (not in this example)

Why generators are interesting?

- They are the smallest sets of items that describe a set of transactions.
  e.g. the smallest sets of products common to a group of customers.
- Some studies have shown shown that generator patterns can provide better accuracy for classification than maximal or closed itemsets.

Why generators are interesting?

- Also, if we find **generator** and **closed** itemsets, we can use them to generate compact representations of association rules of the form:

generator → closed itemset - generator

{cake} → {pasta, orange}

support: 2
confidence: 100%

generator: {cake}
closed: {pasta,orange,cake}

See the paper about IGB assocation rules for details

How to find the generator itemsets?

- **Naïve approach**:
  - by post-processing
  - **inefficient!**
- **Efficient algorithms:**
  - Pascal, DefMe, etc.
  - Find generator itemsets by avoiding generating all frequent itemsets.
  - **Key search space pruning property:** An itemset can only be a generator if all its subsets are also generators.

References – maximal itemsets

FPMax

- *Grahne, G., & Zhu, J. (2003, May). High performance mining of maximal frequent itemsets. In 6th International Workshop on High Performance Data Mining.*

LCM_Max

- *Grahne, G., & Zhu, J. (2003, May). High performance mining of maximal frequent itemsets. In 6th International Workshop on High Performance Data Mining.*

References – closed itemsets

AprioriClose (Close)

- *Nicolas Pasquier, Yves Bastide, Rafik Taouil, Lotfi Lakhal: Discovering Frequent Closed Itemsets for Association Rules. ICDT 1999: 398-416*

Charm

- Mohammed Javeed Zaki, Ching-Jiu Hsiao: CHARM: An Efficient Algorithm for Closed Itemset Mining. SDM 2002.

References – generator itemsets

DefMe

- *Arnaud Soulet, François Rioult (2014). Efficiently Depth-First Minimal Pattern Mining. PAKDD (1) 2014: 28-39*

Pascal

- Bastide, Y., Taouil, R., Pasquier, N., Stumme, G., & Lakhal, L. (2000). Mining frequent patterns with counting inference. *ACM SIGKDD Explorations Newsletter*, 2(2), 66-75.

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